Ensembles-Based and GA-Based Optimization for Landfill Gas Production

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The efficient and economic production of landfill gases (LFG) by optimally adjusting LFG production settings is of high interest as a promising source of biomass energy. A key obstacle in LFG production optimization is the large-scale and complex system with overwhelming uncertainty and heterogeneity. We propose a simplified ensemble-based optimization (EnOpt) method to solve the LFG production optimization problem when constraints are not a concern, where the gradient information is obtained from an ensemble of realizations of the system. For constrained optimization, a novel parameterless genetic algorithm is proposed and successfully applied to the simulated LFG process. The effectiveness of the proposed (EnOpt) method and the parameterless genetic algorithm is demonstrated with the simulation of a landfill and gas generation and transport therein, using a parallel computation strategy. © 2014 American Institute of Chemical Engineers *AIChE J*, 60: 2063–2071, 2014

Keywords: landfill gas system, production optimization, ensemble-based optimization, parallel computation

Introduction

As a result of global economic growth in the past few decades, the consumption of fossil fuels has reached historical high which accelerates the depletion of the world's oil resources. Meanwhile, alternative and renewable energy resources are sought to meet the high demand of energy. The biomass in landfills can generate a large amount of landfill gas (LFG), which consists mainly of CH_4 and CO_2. The LFG is a source of greenhouse gas if released to the atmosphere without control, but can also be a promising source of green energy with proper management and control of its production.

It is challenging but important in modern landfills to produce LFG efficiently and economically due to the large scale and a large number of the systems. It is desirable to maximize the production of LFG by smartly adjusting landfill settings while meeting the safety and environmental regulations. However, little work has been found in the literature that deals with LFG production optimization. In our previous work, we employed artificial neural networks and genetic algorithms (GA) to do forecasting based on historical production data[1] and used ensemble Kalman filter (EnKF) to estimate physical parameters distribution with limited production data.[2] In this article, we explore promising techniques for the production optimization of oil reservoirs and apply them to solve the problem of production optimization in LFG production. Specifically, the production optimization can provide a way to maximize the cumulative production through controlling many critical factors, such as the placement of wells. Similar to a reservoir system, a landfill is a large three-dimensional (3-D) and reactive porous medium. Therefore, the novelty of this article is to apply techniques developed in reservoir engineering to increase the recovery of LFG, which, to our knowledge, has never been used in the production optimization of LFG.

The problem of production optimization can be formulated as maximizing an economic objective function $f(\mathbf{x})$, for example net present value (NPV), in a system by manipulating $\mathbf{x}$. Here, $\mathbf{x}$ can be regarded as a set of control settings, such as the well placement in a landfill, the design of the landfill cover, and the vacuum pressure of extraction wells. A variety of optimization techniques have been proposed and widely used in reservoir engineering. The existing optimization methods can be classified into two major categories, that is, gradient-based methods and gradient-free methods.

The gradient-based methods, including the steepest ascent method, quasi-Newton method,[3] and sequential quadratic programming (SQP),[4] require the gradient of the objective function with respect to the control variables. The adjoint-based method, sensitivity method, and finite difference method are typically used to compute the gradient. The adjoint-based method obtains the gradient by solving an adjoint model using the Jacobian matrix extracted from a simulator. Chen et al.[5] proposed to compute the gradients for history matching and tested its practical use in a single-phase reservoir model. The method was shown to be effective and computationally efficient, compared to the standard constant-zone methods. For multiphase systems, Wu et al.[6] developed a discrete adjoint method for computing sensitivity coefficients in a two-phase flow system. Their method

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could generate the sensitivity of production data to model parameters, which can facilitate the application of the fast convergent Gaussian–Newton algorithm. Li et al.[7] presented a general formulation of calculating the sensitivity matrix for a fully implicit three-phase flow system. They also found that the coefficient matrix was simply the transpose of the Jacobian matrix in the Newton–Raphson solution of the implicit simulator. With the known Jacobian matrix, the procedure of deriving adjoint equations can be avoided. Following this idea, Sarma et al.[8] used an adjoint model for efficient calculation of gradients of NPV with respect to well control settings. These gradients were then used in the SQP method to optimize production. The method was demonstrated successfully for production optimization and closed-loop reservoir management.

The adjoint-based method requires detailed knowledge of the process model used inside the process simulator, which is difficult to obtain for complex and large-scale systems. In contrast, sensitivity and finite-difference methods treat the process simulator as a black box and need the output of the simulator only to evaluate the objective function. Jacquard and Jain[9] introduced a method for calculating sensitivity coefficients of a linear system. They applied the method along with the steepest ascent method for history matching in a 2-D single-phase reservoir by assimilating pressure measurements. Wang et al.[10] used SQP with numerical gradients to solve the production optimization problem in a conventional black-oil reservoir simulator. In their work, the gradient information was computed efficiently by finite difference methods estimated by Sparse Nonlinear Optimizer (SNOPT), a software package for solving large-scale optimization problems.

The second category of optimization algorithms are stochastic methods, including simulated annealing (SA)[11] and GA.[12] The SA method has been used for developing optimal operating rules of reservoir systems, such as well placement, operation scheduling, and injection/extraction rates. Teegavarapu and Simonovic[13] investigated and evaluated the effectiveness of SA for the operation optimization of multi-reservoir systems. They showed that SA could be used for obtaining near-optimal solutions for multiperiod reservoir operation problems that are computationally intractable for conventional methods. Bangerth et al.[14] examined the optimal positions of new injection/production wells to maximize the economic benefits of a reservoir. They compared the performance of SA and finite difference methods, and demonstrated that SA is very efficient in finding near optimal solutions of the optimization problem. The main disadvantage of the SA is its incapability in massively parallel implementations. Therefore, for large-scale porous media systems, such as reservoirs and landfills, SA will fail due to the severe computation burden. In contrast, GA is amenable to parallel computation.[2] Therefore, GA is commonly used in industries and has been applied to the well placement problem in petroleum engineering. Harding et al.[15] applied GA to maximize NPV using problem-specific crossover operators. Comparing to the results of SA and SQP, they showed that GA had significant performance improvements. Almeida et al.[16] used GA for the management of intelligent oil fields by selecting optimal valve control configurations. They even considered the existence of uncertainties in valve operations, such as the risk of valve failure. However, being gradient-free, these methods require tens of thousands of evaluations of the objective functions, which can still be time consuming for large-scale and complex systems such as reservoirs and landfills.

To handle the problem of production optimization for reservoir systems, Chen et al.[17] proposed ensemble-based optimization (EnOpt), which can be regarded as a combination of gradient-based methods and stochastic methods. Similar to the EnKF, where the covariance matrix of the state vector is approximated by an ensemble of model realizations, the gradient information of EnOpt is acquired from an ensemble of control scenarios. The advantage of EnOpt is that it treats the simulator as a black box, so that the model equations inside the simulator are not required. After the gradient information was obtained, Chen et al.[17] used the steepest ascent method to update the control variables by iteration. The applicability of EnOpt was assessed through a synthetic reservoir example, where a significant increase in NPV was observed. Su and Aramco[18] showed the application of EnOpt on a large-scale reservoir model with 200,000 cells. Even for such a complex system, the cumulative water production was reduced by 50% after a few iterations. Leeuwenburgh et al.[19] discussed several aspects of EnOpt, including the ensemble size, perturbation, regularization, and smoothing, by comparing it to the adjoint approach. They also investigated the effect of the time scale on the optimization performance for the well placement problem. EnOpt was also employed on the capacitance resistive model to efficiently optimize large water flooding operations in Jafroodi and Zhang[20]. Their simulation result showed that the optimal injection rates could achieve a significantly higher oil production and an improved reservoir sweep efficiency. All these efforts show that EnOpt can work efficiently in the optimization of large-scale reservoir systems. To improve the convergence, Chaudhri et al.[21] developed a conjugate gradient EnOpt (CGEnOpt). This method was applied to a synthetic 3-D reservoir model with improvement convergence rate.

In this article, we first briefly describe the formulation of the LFG production optimization problem in the landfill system. Then, we give a review of EnOpt and CGEnOpt methods proposed by Chen et al.[17] and Chaudhri et al.[21], respectively. Next, EnOpt is interpreted from another aspect, which further simplifies the algorithm. Relevant issues concerning the application of EnOpt, such as step size selection, parallel computation, and implementation procedure, are also discussed. The methods of GA, EnOpt, and CGEnOpt for the production optimization of a simulated landfill system without constraints are then demonstrated. By comparing the optimization performance for these algorithms, we show that all the methods can achieve a significant increase of NPV, while CGEnOpt is shown to achieve the best convergence. For constrained production optimization, we first look into the existing solutions and then propose a novel parameterless GA algorithm. The applicability of the proposed parameterless GA is tested and evaluated on the same landfill system with explicit constraints. The last section presents conclusions of the article.

The LFG Production Optimization Problem

For model-based optimization, the first step is to define a simulation model to describe the problem. Here, we use a comprehensive 3-D model of gas generation and transport developed earlier by the authors,[1,2,22–25,] which has been

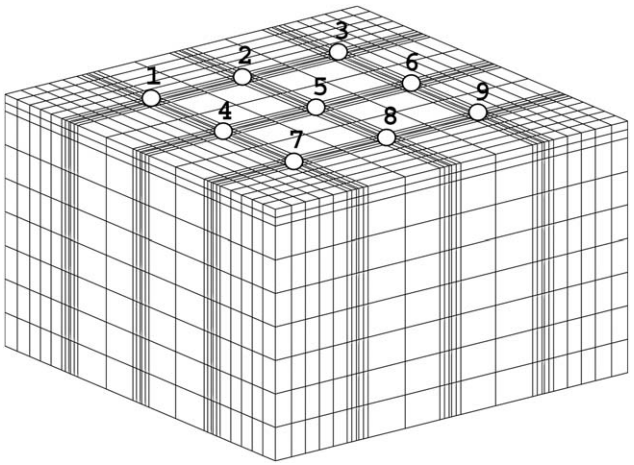

Figure 1. Three-dimensional view of the grid structure used in the computations.

The locations and numbering of the nine extraction wells are shown by the circles.

validated by real landfill data and has proven to be an accurate model. The model is based on the convection-diffusion-reaction equation, given by

$$\epsilon_\ell \frac{\partial \rho_k}{\partial t} + \nabla \cdot (\rho_k \mathbf{V}) = \alpha_k(Z) + \nabla \cdot (D_{e_{km}} \nabla \rho_k) \tag{1}$$

Detail of the model and the numerical scheme for solving Eq. 1 are well-documented in Hashemi et al.[22] and Sanchez et al.,[23–25] which will not be repeated here to save space.

The physical size $L_x \times L_y \times L_z$ of the model landfill is assumed to be $30 \times 30 \times 30\,\mathrm{m}^3$, represented by a computational grid of $57 \times 57 \times 37$ blocks, where z denotes the vertical direction. The cover's thickness is assumed to be 2 m with an isotropic permeability of 0.1 mD. The landfill walls are assumed impermeable. Figure 1 depicts a 3-D view of the structure of the computational grid that represents the landfill. The landfill model is assumed to contain nine symmetrically located extraction wells (see Figure 1). Due to the limited access to the real landfill, the landfill model is subject to high geological uncertainty, which is permeability distribution in our case. Fortunately, we are able to estimate permeability from the measurement data by performing real-time history matching using EnKF.[2] Therefore, we can use the updated permeability distribution from EnKF to deal with the geological uncertainty. Figure 2 depicts the final permeability distribution obtained from real-time estimates using EnKF. Figure 3 presents the permeability distribution of a horizontal layer of the reference system at a depth of 15 m from the top.

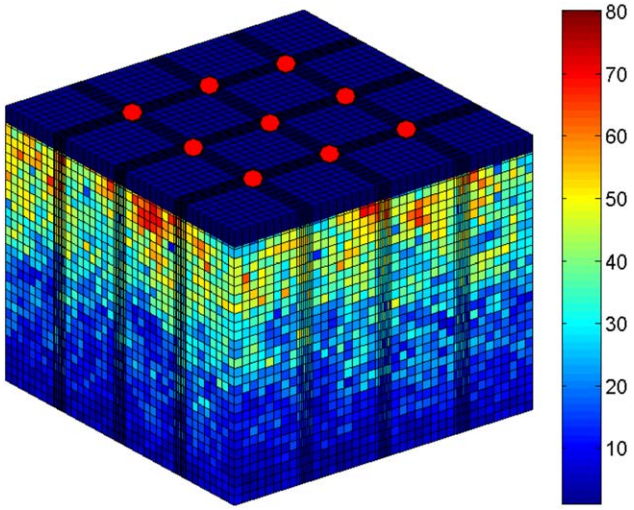

Figure 2. Three-dimensional view of the grid structure and the reference permeability distribution.

The permeability is in mD. The location of nine extraction wells are shown by the circles. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

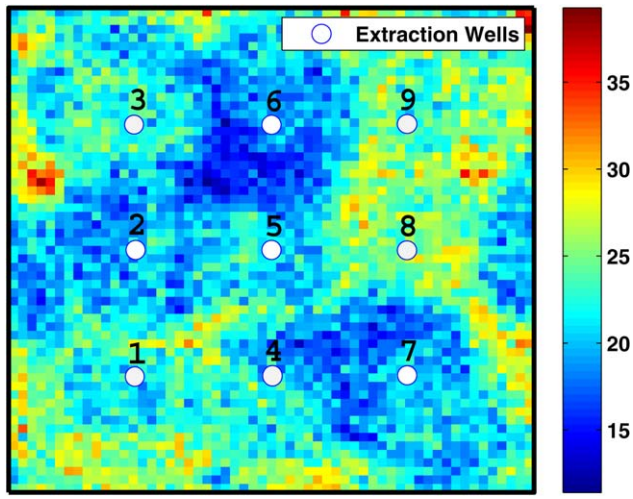

Figure 3. The spatial distribution of estimated permeability in a horizontal layer at a depth of 15 m.

The permeability is in mD. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

We define $\mathbf{x}$ as the vector of control variables, which are vacuum pressures in the extraction wells, as follows

$$\mathbf{x} = (x_1, x_2, \ldots, x_{N_x}), \tag{2}$$

where N_x, the total number of control variables and also the number of extraction wells, is nine in the current landfill model.

The objective function to maximize is the NPV over a period of 2 years, which is defined as follows

$$g(\mathbf{x}) = \sum_{i=1}^{N_x} \left[\sum_{j=1}^{N_t} \left[f_i^j(\mathbf{x}) \tau^j \right] - h_i(\mathbf{x}_i) \right], \tag{3}$$

where N_t is the total number of time steps, N_x is the number of extraction wells, $f_i^j(\mathbf{x})$ is the cumulative production of CH_4 for well i over time step $\triangle t_j$, τ^j is the price of natural gas over time step $\triangle t_j$, and $h_i(\mathbf{x}_i)$ is the operational cost associated with the vacuum pressure in the ith extraction well. Figure 4 depicts the predicted gas price for the next 2 years based on the historical gas price for the past 30 years. Figure 5 presents a simple profile of $h_i(\mathbf{x}_i)$ with respect to the vacuum pressure in an extraction well.

Production optimization in landfills is an unconstrained problem when the only objective is to maximize production. When constraints have to be considered, such as constraints on the production rates, it is a constrained problem. For the constrained case, we consider constraining the maximum

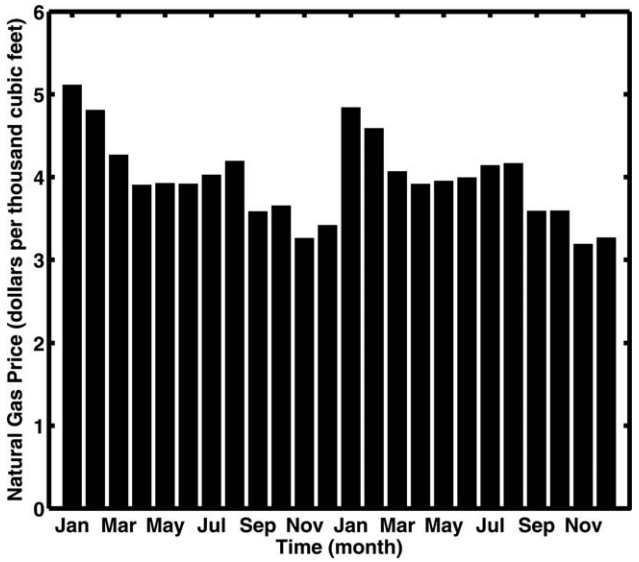

Figure 4. The predicted natural gas price for the next 2 years.

flow rate for extraction wells. This is a common and reasonable constraint due to the limited capacity of transportation and treatment.

For the constrained optimization with the maximum flow rate constraint, the production optimization can be formulated as

$$\max_{\mathbf{x}} g(\mathbf{x}) \quad \text{subject to} \quad \text{flow}_i(\mathbf{x}) < \rho, i=1,2,\ldots,N_x \tag{4}$$

where $g(\mathbf{x})$ is the NPV defined in Eq. 3, $\text{flow}_i(\mathbf{x})$ is the flow rate for the ith extraction well, and ρ is the maximum flow rate, which is assumed to be the same for all the extraction wells.

The inclusion of the flow rate constraints will adversely lead to additional computation cost. The operation of landfill plants often requires prompt solutions due to frequently changing environmental factors around landfills. In this case, the constrained optimization can be too slow to come up with an optimal or even feasible solution. Therefore, in the following sections, we will discuss unconstrained optimization and constrained optimization separately using different approaches. The ensemble-based algorithms, EnOpt and CGEnOpt, are designed for the unconstrained optimization. LFG production operators can choose an appropriate method to handle either unconstrained and constrained cases based on the requirement of the real production situation.

Unconstrained Production Optimization

Review of existing EnOpt methods

In this section, we start with reviewing the general procedures of EnOpt and CGEnOpt. In EnOpt, Chen et al.[17] utilized the steepest ascent method to update the control variables $\mathbf{x}$

$$\mathbf{x}_{l+1} = \frac{1}{\alpha_l}\mathbf{C}_x\mathbf{G}_l^T + \mathbf{x}_l, \tag{5}$$

where l denotes the iteration index, α_l is a tuning parameter that determines the step size in the steepest ascent direction and is calculated by a line search method, $\mathbf{G}_l$ is the sensitivity of $g(\mathbf{x})$ to the control variables $\mathbf{x}$ at the lth iteration step, and $\mathbf{C}_x$ is the smoothing matrix. Chen et al.[17] proved that $\mathbf{C}_x\mathbf{G}_l^T$ can be approximated by $\mathbf{C}_{\mathbf{x},g(\mathbf{x})}$, which is defined by

$$\mathbf{C}_{\mathbf{x},g(\mathbf{x})} = \frac{1}{N_e-1}\sum_{j=1}^{N_e}(\mathbf{x}_{l,j}-\langle\mathbf{x}_l\rangle)(g(\mathbf{x}_{l,j})-\langle g(\mathbf{x}_l)\rangle)), \tag{6}$$

where

$$\langle\mathbf{x}_l\rangle = \frac{1}{N_e}\sum_{j=1}^{N_e}\mathbf{x}_{l,j}, \tag{7}$$

and

$$\langle g(\mathbf{x}_l)\rangle = \frac{1}{N_e}\sum_{j=1}^{N_e} g(\mathbf{x}_{l,j}), \tag{8}$$

where N_e denotes the ensemble size, $\mathbf{x}_{l,j}$ is the jth ensemble member of control variables generated by perturbing $\mathbf{x}_l$ with zero mean Gaussian random noise, and $g(\mathbf{x}_{l,j})$ is the corresponding NPV for the jth ensemble. Because the Gaussian noise is with a zero mean, we have $\langle\mathbf{x}_l\rangle \approx \mathbf{x}_l$ and $\langle g(\mathbf{x}_l)\rangle \approx g(\mathbf{x}_l)$. Therefore, the product of $\mathbf{C}_x\mathbf{G}_l^T$ can be approximated by

$$\mathbf{C}_{\mathbf{x},g(\mathbf{x})} \approx \mathbf{C}_x\mathbf{G}_l^T \tag{9}$$

With this approximation, the gradient can be estimated from the ensemble cross covariance. Therefore, the steepest ascent method in EnOpt becomes

$$\mathbf{x}_{l+1} = \frac{1}{\alpha_l}\mathbf{C}_x\mathbf{C}_{\mathbf{x},g(\mathbf{x})} + \mathbf{x}_l \tag{10}$$

To improve the convergence, Chaudhri et al.[21] developed the CGEnOpt, where the search direction was determined by a conjugate direction. By using the linear conjugate method, the steepest ascent method becomes

$$\mathbf{x}_{l+1} = \frac{1}{\alpha_l}\left[\mathbf{C}_x\mathbf{G}_l^T + \beta_l\mathbf{C}_x\mathbf{G}_{l-1}^T\right] + \mathbf{x}_l, \tag{11}$$

where β is determined by the Fletcher–Reeves formula

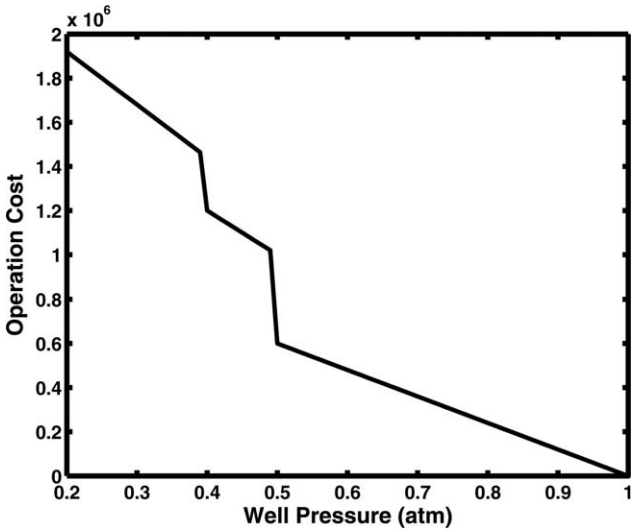

Figure 5. Function of operation cost $h_i(\mathbf{x}_i)$ with respect to the vacuum pressure $\mathbf{x}_i$ of an extraction well.

$$\beta=\frac{(\mathbf{C}_x\mathbf{G}_l^T)(\mathbf{C}_x\mathbf{G}_l)}{(\mathbf{C}_x\mathbf{G}_{l-1}^T)(\mathbf{C}_x\mathbf{G}_{l-1})} \tag{12}$$

Interpretation of EnOpt for small ensemble sizes

The approximation by Eq. 9 will be satisfied only under the condition of zero mean Gaussian perturbation. For a small ensemble size, this is usually not appropriate. Therefore, we interpret EnOpt from an aspect of multiple variable Taylor expansion.

First, we can define the vector of ith ensemble member $\mathbf{x}_{l,i}$ as

$$\mathbf{x}_{l,i}=(x_{l,i}^1, x_{l,i}^2, \ldots, x_{l,i}^{N_x}) \tag{13}$$

where $\mathbf{x}_{l,i}^j$ denotes the vacuum pressure of jth extraction well for the ith ensemble member at the lth iteration step.

The current vector of control variables $\mathbf{x}_l$ is defined as

$$\mathbf{x}_l=(x_l^1, x_l^2, \ldots, x_l^{N_x}) \tag{14}$$

and the gradient $\mathbf{G}_l$ is

$$\mathbf{G}_l=(G_l^1, G_l^2, \ldots, G_l^{N_x}) \tag{15}$$

From Taylor expansion, we know that the NPV of ith ensemble ($g(\mathbf{x}_{l,i})$) can be linearized at ith ensemble x_l as

$$\begin{aligned} g(\mathbf{x}_{l,i}) &\approx g(\mathbf{x}_l)+G_l^1(x_{l,i}^1-x_l^1)+G_l^2(x_{l,i}^2-x_l^2)+\cdots+G_l^{N_x} \\ &\times(x_{l,i}^{N_x}-x_l^{N_x})=g(\mathbf{x}_l)+G_l^1\Delta x_i^1+G_l^2\Delta x_i^2+\cdots+G_l^{N_x}\Delta x_i^{N_x} \end{aligned} \tag{16}$$

which can be rewritten in the matrix form

$$\begin{aligned} \mathbf{g}_e &= \begin{bmatrix} g(\mathbf{x}_{l,1}) \\ g(\mathbf{x}_{l,2}) \\ \vdots \\ g(\mathbf{x}_{l,N_e}) \end{bmatrix} = \begin{bmatrix} g(\mathbf{x}_l) \\ g(\mathbf{x}_l) \\ \vdots \\ g(\mathbf{x}_l) \end{bmatrix} \\ &+ \begin{bmatrix} G_l^1 G_l^2 \cdots G_l^{N_x} \end{bmatrix} \begin{bmatrix} \Delta x_1^1 \Delta x_2^1 \cdots \Delta x_{N_e}^1 \\ \Delta x_1^2 \Delta x_2^2 \cdots \Delta x_{N_e}^2 \\ \vdots \\ \Delta x_1^{N_x} \Delta x_2^{N_x} \cdots \Delta x_{N_e}^{N_x} \end{bmatrix} = \mathbf{g}_0+\mathbf{G_l}^T\mathbf{A} \end{aligned} \tag{17}$$

We then use the least-square method to solve the above equation

$$\mathbf{G}_l=(\mathbf{A}\mathbf{A}^\mathbf{T})^{-1}\mathbf{A}(\mathbf{g_e}-\mathbf{g_0})^T \tag{18}$$

From the above derivations, we can see that the alternative interpretation of the EnOpt method makes it applicable to the non-Gaussian case. Besides, it has the same advantage as the original EnOpt, which is suitable for parallel computing. After obtaining the gradient information, we combine it with the steepest ascent or conjugate gradient method to perform the optimization.

Step size selection

To produce a reliable and robust production optimization using EnOpt, we must also choose an appropriate line search method to determine the step size α_l. An optimal step size $\hat{\alpha}_l$ needs to satisfy the following rule

$$\hat{\alpha}_l=\min_\alpha \theta(\alpha)=\min_\alpha g\left(\frac{1}{\alpha}\mathbf{G}_l+\mathbf{x}_l\right) \tag{19}$$

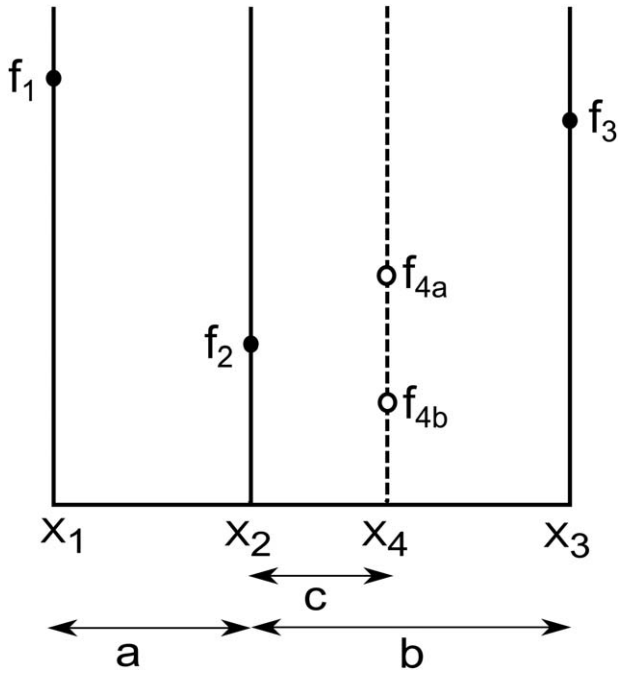

Figure 6. Diagram to show golden section search technique.

However, it is difficult to find the optimal step size that exactly satisfies Eq. 19. In general, we need an iterative method to find a near optimal step size. In literature, there are many methods for solving this problem, such as the binary search method, the Cauchy search method, and the golden section search method. The binary search method is the simplest to handle the line search problem, but its low efficiency is an issue. That is why the golden section search method is proposed, which has been proven to converge faster than the binary search method in the literature. Moreover, compared with the golden search method, the Cauchy search method involves additional steps to determine Cauchy parameters, which takes more computation time. Considering that time efficiency is critical for landfill problems, we choose the golden section search method in this article.

The golden section search method is a technique for finding the extremum of a unimodal function by iteratively narrowing the range of values inside which the minimum/maximum is known to exist. Figure 6 illustrates a single step of the technique to find the minimum of a function $f(x)$. The horizontal direction is the x parameter and vertical direction is the value of $f(x)$. Suppose that we already evaluate the function values at x_1, x_2, and x_3, which are denoted as f_1, f_2, and f_3, respectively. For a unimodal function $f(x)$, if f_2 is smaller than f_1 and f_3, it can be proved that the minimum lies between x_1 and x_3. Now, we have three points x_1, x_2, and x_3, which form a triplet of points. For the next step in the golden section search method, we have to evaluate the $f(x)$ at a new point x_4. An efficient selection of x_4, the interval c in this example, can be derived mathematically as follows[26]

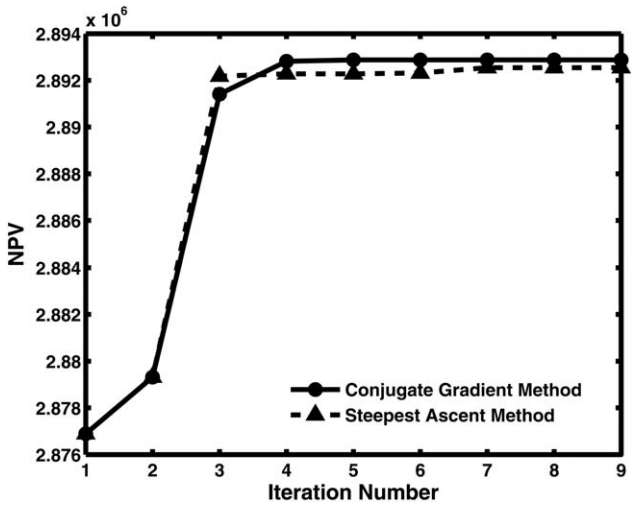

Figure 7. Evolution of NPV with the iteration steps for conjugate gradient method (solid line) and steepest ascent method (dotted line).

$$\frac{b}{a}=\frac{b-c}{c} \tag{20}$$

where the ratio of $\frac{b}{a}$ is the golden ratio 1.618.

If the evaluation of $f(x)$ at the new point x_4 yields f_{4a}, the new triplet of points is x_1, x_2, and x_4. Similarly, the new triplet of points is x_2, x_4, and x_3 for the case of f_{4b}.

Parallel implementation of EnOpt and CGEnOpt

The aforementioned landfill problem involves a large computational grid that requires extensive computations. To implement the EnOpt, we must solve the governing equations of gas generation and transport a large number of iterations to evaluate the objective function for each ensemble. The advantage of the EnOpt is that the calculation of NPV for each member of the ensemble is independent of those for other ensemble members, as each ensemble requires only a solution of the governing equation. Hence, the EnOpt implementation is amenable to parallel computing.

A master–slave parallel computational strategy is employed in [24] for EnOpt, with the slave processors evaluating the NPV for each ensemble, and the master processor collecting all the NPV evaluations to perform gradient-based optimization. The message-passing interface was the platform used for parallel computing. The number of the slave processors used in the EnOpt was 64, which is the same as the ensemble size. The procedure adopted in this article is summarized as follows.

1. At the first time step $l=1$, generate the initial state vector of control variables $\mathbf{x}_l$, which is sampled from a uniform distribution with lower and upper limits as 0.2 and 1 atm, respectively. At other time steps $l=1, \mathbf{x}_l$ is set equal to the best guess at the previous step $l-1$. Then an ensemble of control variables $\mathbf{x}_{l,i}, i=1,2,\ldots,N_e$ is generated by perturbing $\mathbf{x}_l$ with Gaussian noise with zero mean and a standard deviation that is a specified percentage (10 percent in this work) of the well pressure $\mathbf{x}_l$.
2. Run the landfill simulator for the given period to evaluate the NPV $g(\mathbf{x}_{l,i})$ for each ensemble member using parallel computing described in Section Parallel implementation of EnOpt and CGEnOpt.
3. Compute the gradient vector $\mathbf{G}_l$ using the least-square method described in Section Interpretation of EnOpt for small ensemble sizes.
4. Determine the optimal step size α_l using the golden section search method.
5. Update the state vector $\mathbf{x}_{l+1}$ using either steepest ascent method or conjugate gradient method. At the same time, evaluate the NPV $g(\mathbf{x}_{l+1})$ for the updated state vector $\mathbf{x}_{l+1}$.
6. The stopping criteria are $\frac{g(\mathbf{x}_{l+1})-g(\mathbf{x}_l)}{g(\mathbf{x}_l)}<\varepsilon$ and $\frac{||\mathbf{x}_{l+1}-\mathbf{x}_l||}{||\mathbf{x}_l||}<\eta$, where ε and η are tolerances for $g(\mathbf{x}_l)$ and $\mathbf{x}_l$, respectively. If the stopping criteria are satisfied, terminate the optimization; otherwise set $l=l+1$ and go back to Step 1.

Simulation results

We evaluate the performance of EnOpt methods for the landfill system described in Section The LFG Production Optimization Problem. The main objective of this optimization problem is to find the optimal pressure settings of extraction wells to maximize the NPV for a period of 2 years. Figure 7 shows the evolutions of NPV obtained from the steepest ascent method and the conjugate gradient method. Both methods can achieve significant increase of NPV within nine iteration steps. Since each evaluation of NPV takes almost 1 h by using parallel computation on the Linux cluster at University of Southern California, both approaches finish the optimization within approximately 10 h. For a typical landfill, it is under dynamic changes in terms of system properties, such as permeability, porosity, and tortuosity. Other changes that can happen to landfills are weather conditions and LFG extractions. These dynamics will affect the landfill model and thus the optimal strategy needs to be updated over time. Since EnOpt methods take less than 10 h to compute, a daily update scheme could be implemented for the landfill production optimization.

In terms of the optimization performance, the conjugate gradient method converges after the fifth iteration step and its maximum NPV is higher than that of the steepest ascent method. Therefore, conjugate gradient method is relatively more efficient for this synthetic landfill optimization problem, which has also been validated by Chaudhri et al.[21] for a reservoir system. Figure 8 depicts the optimal well pressure

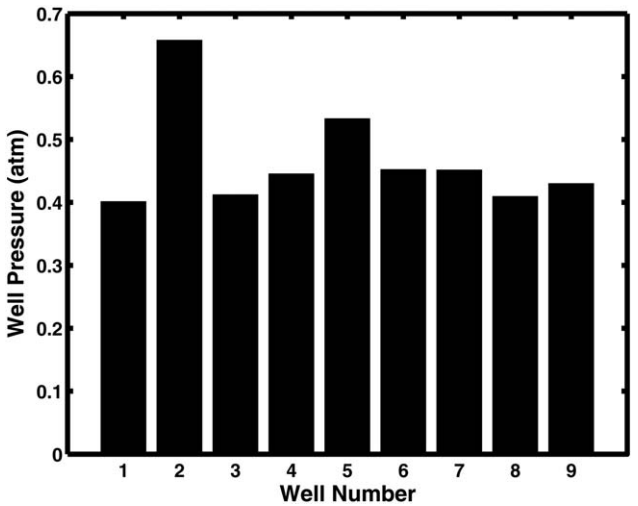

Figure 8. The optimal well pressure for nine extraction wells obtained from the CGEnOpt.

for each extraction well-obtained from the conjugate gradient method. The optimal well pressures for all the extraction wells are within 0.4–0.7 atm. The reason is that both the flow rate and the operation cost will increase when the vacuum pressure decrease. Therefore, the optimal well pressure will be in the middle of the range, being 0.4–0.7 atm in this case. Moreover, as the landfill system is a highly heterogeneous porous medium with the physical properties spatially distributed, the optimal vacuum pressures are different for each extraction well.

Considering that the landfill system is a large-scale system, the parallel GA can also be used to perform the optimization. Aside from the computation cost, GA is believed to be very efficient in finding the optimal solution within a specified search region. Similar to CGEnOpt, GA only needs the evaluation of the objective function, which is independent of the species. This makes GA amenable to parallel computing. To be comparable to CGEnOpt, we set the number of species in GA to be 64, same as the ensemble size used in CGEnOpt. Therefore, for each iteration, both GA and CGEnOpt need to evaluate NPV for 64 ensemble members. Moreover, CGEnOpt needs additional evaluations of NPV, normally fewer than 10 evaluations, to determine the optimal step size. Figure 9 compares the optimization results of GA and CGEnOpt. As indicated by the result, GA can achieve the same maximum NPV value as CGEnOpt does, but needs 25 iterations to converge. Therefore, even taking the computation of deciding optimal step size into account, it is evident that CGEnOpt is computationally more efficient than GA.

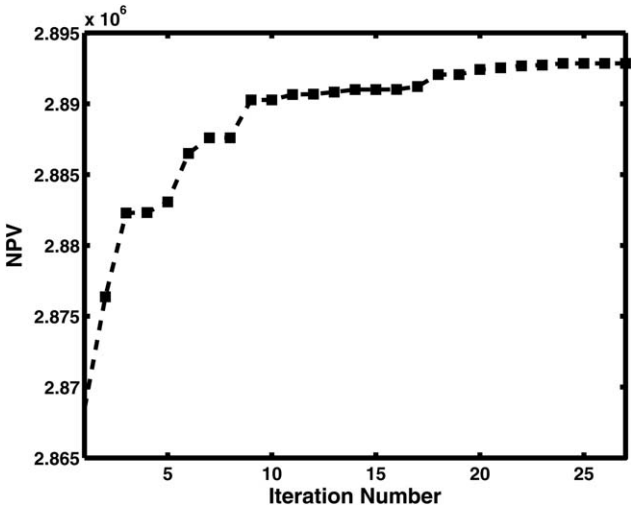

Figure 9. Evolution of NPV with iterations for the GA method.

Production Optimization with Constraints

Traditional constrained optimization methods

The existing ensemble-based methods, EnOpt and CGEnOpt, cannot handle nonlinear constraints. Usually, constrained optimization can be converted to unconstrained optimization through several techniques, such as the penalty function method, the augmented Lagrange multiplier method, quadratic programming, and the gradient projection method. Here, we introduce two popular techniques to solve the constrained optimization.

The penalty function method as described in Snyman[27] is as follows

$$\max_{\mathbf{x}} \left[g(\mathbf{x}) - \sum_{i=1}^{N_x} \beta_i (\text{flow}_i(\mathbf{x}) - \rho)^2 \right] \tag{21}$$

The penalty parameter β_i is given by

$$\beta_i = \begin{cases} 0 & \text{if } \text{flow}_i(\mathbf{x}) < \rho \\ \gg 0 & \text{if } \text{flow}_i(\mathbf{x}) \geq \rho \end{cases} \tag{22}$$

We need to find an appropriate value of β_i, because a large value of β_i can cause instability when deriving a solution with high accuracy. The sequential unconstrained minimization technique, which incrementally increases the penalty parameter, can be used to handle this.

The augmented Lagrange method is proposed to solve for inequality constraints based on the traditional Lagrange method. One possible augmented Lagrangian function is given by[27]

$$\max_{\mathbf{x}} \left\{ g(\mathbf{x}) - \sum_{i=1}^{N_x} \left[\max \left(\frac{1}{2} \lambda_i + \beta (\text{flow}_i(\mathbf{x}) - \rho), 0 \right) \right]^2 \right\} \tag{23}$$

where λ_i is the Lagrange multiplier and β is an adjustable penalty parameter. Similar to the penalty function, these parameters need to be determined by an iterative approximation.

For both penalty function method and augmented Lagrange method, we have to perform the EnOpt with the modified objective functions (Eqs. 21 and 23) iteratively, to obtain proper parameter values. This can be really time-consuming for large-scale landfill systems, where each evaluation of NPV needs almost 1 h. Due to the computation cost in estimating the optimal parameters, these EnOpt methods could fail.

Parameterless GA

To avoid the potentially high computation cost of the ensemble-based methods, we propose a parameterless GA method which is actually a penalty method. The parameterless GA method here is different from the GA method we used in one of our previous papers,[1] which does not involve any constraints. A unique feature of the parameterless GA is that it does not need to estimate optimization parameters.

GA are based on the concept from genetics to describe the individual and Darwinian evolution theory to yield better solutions of the problem. In general, the use of the GA involves three steps.[28]

1. The selection step for choosing the "species" (estimates of the parameters—the extraction wells' pressure in our study) that generate the offsprings (new estimates).
2. The crossover and mutation steps that generate new species that improve the optimal estimates.
3. The eliticism step to select species that lead eventually to the global minimum of the objective function, NPV in our work.

In the selection step, the species with a larger objective function value (i.e., NPV in this work) has a higher possibility to be selected to produce offsprings in the crossover step. For the selection of species, three common techniques are used in GA: proportionate selection, ranking selection, and tournament selection. The one utilized in this work is the binary tournament.[24] We first set a prior probability p_s of selection. Then, we pick two random species from the current generation. A random number r, uniformly distributed

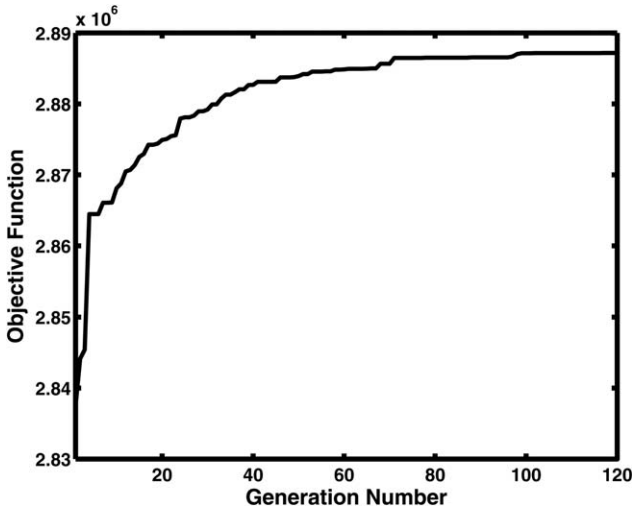

Figure 10. Evolution of NPV with iterations using the parameterless GA.

in (0,1), is generated. If $r < p_s$, we select species with a larger NPV. Otherwise, a species with a smaller NPV is selected. After repeating the same procedure multiple steps, we are able to choose enough species for the crossover step. In this work, we set $p_s=0.85$, large enough to ensure an optimal solution. For the parameterless GA, we modify the selection method by enforcing the following criterion, that is, any species satisfying the constraints is preferred over any species that violates any constraint. This can be accomplished by modifying the objective function of GA as follows

$$\max_{\mathbf{x}} g(\mathbf{x}) = \begin{cases} \sum_{i=1}^{N_x} \left[\sum_{j=1}^{N_t} \left[f_i^j(\mathbf{x}) \tau^j \right] - h_i(\mathbf{x}) \right] & \text{if } \text{flow}_i(\mathbf{x}) < \rho \\ 0 & \text{if } \text{flow}_i(\mathbf{x}) \geq \rho \end{cases} \tag{24}$$

From the modified objective function, we can see that if a species violates any constraint, it has a small possibility to be chosen due to the zero objective function.

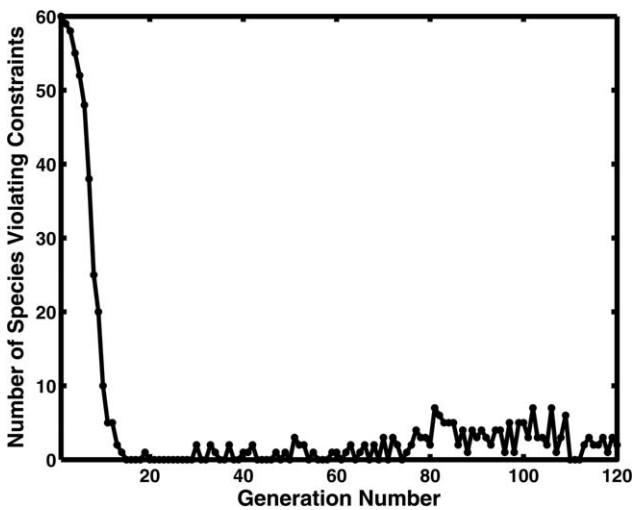

Figure 11. Evolution of number of species violating constraints with generations in the parameterless GA.

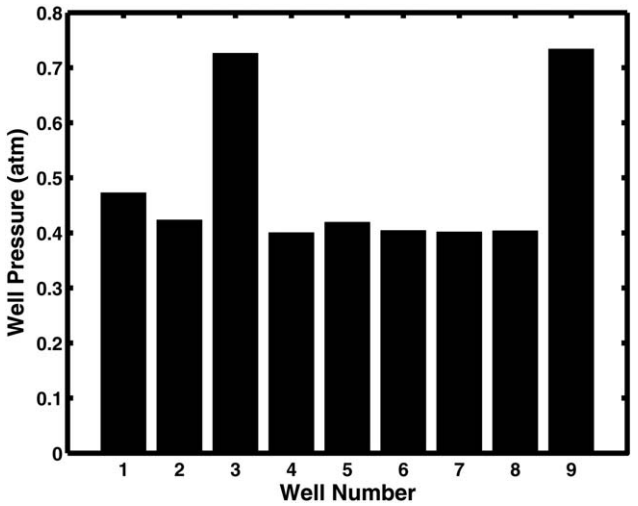

Figure 12. The optimal well pressure for nine extraction wells obtained from the parameterless GA.

Simulation results

The effectiveness of the parameterless GA method is demonstrated on the same landfill system used in Section Unconstrained Production Optimization. As illustrated by the results in Figure 10, the objective function converges to the maximum NPV in 120 iterations. Figure 11 shows the evolution of the number of species that violate constraints at each generation. As iterations proceed, the number of violations decreases significantly. At the end of optimization, there are still several species, fewer than five, violating constraints, which is acceptable in GA. The reason is that the crossover and mutation step can generate species that violate constraints, although the selection step tends to choose a species satisfying constraints. In spite of this phenomenon, the optimal species do not violate constraints, because its objection function is not zero. Figure 12 shows the optimal well

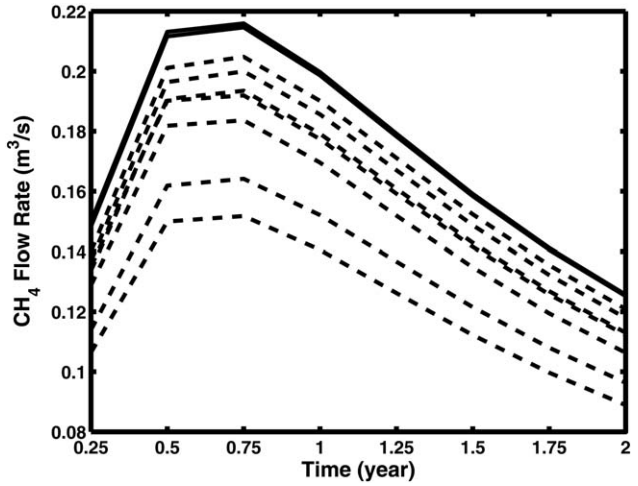

Figure 13. The CH_4 flow rate of all extraction wells with the same vacuum pressure of 0.5 atm: (solid lines) flow rate of Wells 3 and 9; (dotted lines) flow rate of other extraction wells.

pressure distribution from the parameterless GA. Compared to Figure 8, Wells 3 and 9 have higher well pressure for the constrained optimization. The reason is that the gas flow rates of Wells 3 and 9 are larger than those of other wells with the same vacuum pressure (see Figure 13). Typically, a smaller vacuum pressure in an extraction well will yield a higher flow rate. Therefore, with the same flow rate constraint, the optimal well pressures for these two wells will be higher. This phenomenon could be explained by the heterogeneous permeability distribution (see Figure 3). Because the permeability around Wells 3 and 9 is relatively high, the gas flow rate of these two wells tends to be higher.

Conclusions

In this article, we present a simplified EnOpt method and a parameterless GA method for unconstrained and constrained LFG production optimization, respectively. Relevant issues concerning the application of EnOpt, such as step size selection, parallel computing technique, and the implementation procedure, are investigated successfully. Test results on a simulated large-scale landfill optimization problem show the effectiveness and efficiency of the proposed EnOpt, which is implemented via parallel computing for critical real-time production optimization. Comparison of the performance of different optimization methods, including the steepest ascent method, the conjugate gradient method, and the GA indicates that the conjugate gradient method is able to achieve a higher NPV with faster computation. The proposed parameterless GA method is successfully applied to the landfill optimization problem with constraints on the maximum flow rates. Future work could benefit from testing the optimization strategies on a real LFG system.

Acknowledgments

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